Innermost Stable Circular Orbits Around Rotating Compact Stars

K. Goluchová[1], G. Török[1], M. Urbanec[1], E. Šrámková[1], G. Urbancová[1, 2]

[1] Research Centre for Computational Physics and Data Processing, Institute of Physics, Faculty of Philosophy and Science, Silesian University in Opava, Bezručovo nám. 13, 746 01 Opava, Czech Republic, e-mail: gabriel.torok@gmail.com

[2] Astronomical Institute, Boční II 1401/2a, CZ-14131 Praha 4 - Spořilov, Czech Republic

www.astrocomp.physics.cz

Silesian University in Opava

INSTITUTE OF PHYSICS SU OPAVA

EUROPEAN UNION

MINISTRY OF EDUCATION, YOUTH AND SPORTS

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Abstract

Orbital motion close to a rotating neutron star (NS) is affected by effects of strong gravity. Keplerian frequency at the innermost stable circular orbit (ISCO frequency) depends on the interplay between relativistic effects and geometric Newtonian effects given by NS oblateness, and may increase as well as decrease when NS angular momentum increases. In this context we examine a large set of NS equations of state.

Behaviour of ISCO frequency

Keplerian frequency at the innermost stable circular orbit, $\nu_{\rm ISCO}$, has been frequently assumed within several astrophysical studies. The relativistic formula linear in NS angular momentum j determining approximative value of $\nu_{\rm ISCO}$ for the rotating NS (Kluzniak & Wagoner, 1985) can be written in the form

$$\nu_{\rm ISCO} = \frac{2200 M_\odot}{M}\left(1+0.75j\right), \tag{1}$$

This formula is for it's simplicity and robustness often recalled and assumed within miscellaneous astrophysical applications.

The ISCO frequency however depends on the interplay between relativistic effects and geometric Newtonian effects given by NS oblateness. For a given gravitational mass M and NS equation of state (EoS), there is roughly linear relation between j and NS rotation frequency $\nu_{\rm S}$, and $\nu_{\rm ISCO}$ may increase as well as decrease when $\nu_{\rm S}$ and j increase. Therefore, there exists maximal value of $\nu_{\rm ISCO}$ which often does not correspond to the maximal allowed $\nu_{\rm S}$.

Terms Quadratic in j

In order to demonstrate main trends in behaviour of ISCO frequency, we use Hartle-Thorne approximation of NS spacetime in its usual form envolving first and second order terms in j and first order terms in the NS quadrupole moment q. The ISCO radius can be here expressed in units of gravitational mass of the compact object as (e.g., Abramowicz et al. 2003)

$$r_{\rm ISCO} = 6\left[1 - j\frac{2}{3}\sqrt{\frac{2}{3}} + j^2\left(\frac{251647}{2592} - 240\ln\frac{3}{2}\right) + q\left(-\frac{9325}{96} + 240\ln\frac{3}{2}\right)\right] \approx 6\left[1 - 0.54\,j - 0.23\,j^2 + 0.18\,q\right]. \tag{2}$$

The ISCO frequency in units of Hertz can be then written in the form

$$\nu_{\rm ISCO} = \frac{2200 M_\odot}{M}\left[1 + \frac{11j}{6^{3/2}} + \frac{1}{864}j^2\left(-160583 + 397710\ln\frac{3}{2}\right) + \frac{5}{32}q\left(1193 - 2946\ln\frac{3}{2}\right)\right] \approx \frac{2200 M_\odot}{M}\left[1 + 0.75\,j + 0.78\,j^2 - 0.23q\right]. \tag{3}$$

In the first two panels of Figure 1 we illustrate dependence of the ISCO frequency $\nu_{\rm ISCO}$ on the NS angular momentum j and the quadrupole moment q. In the same two panels of Figure 1 we include dependency of $\nu_{\rm ISCO}$ on a scaled quadrupole moment $\tilde{q} \equiv q/j^2$. Assuming this quantity we can introduce the quadratic correction to the linear relation (1) given by coefficient

$$\delta \simeq 0.78 - 0.23\tilde{q} \tag{4}$$

and rewrite the equation (3) as

$$\nu_{\rm ISCO} \approx \frac{2200 M_\odot}{M}\left(1 + 0.75j + \delta j^2\right). \tag{5}$$

Maximal ISCO frequency (Hartle-Thorne Spacetimes)

For fixed values of $\tilde{q}$ in relation (5), one can find a maximal frequency $\nu_{\rm ISCO}^{\max}$ ($\partial\nu_{\rm ISCO}/\partial j = 0$) which arises for

$$j_{\max} = -\left(\frac{11\sqrt{6}}{72}\right)\frac{1}{\delta} \approx -\frac{0.375}{\delta}. \tag{6}$$

We illustrate existence and behaviour of $\nu_{\rm ISCO}^{\max}$ in Figure 1b. Physical nature of this non-monotonicity follows from the qualitative difference between influence of relativistic and Newtonian effects (Kluźniak, W., Rosińska, D., 2014) reflected by the decreases of $r_{\rm ISCO}$ with increasing j and its increase with increasing q. As discussed by Török et al. (2014), the function $r_{\rm ISCO}(j)$ has a minimum when j and q are related as follows

$$j_{\min} \approx \frac{0.27}{-0.23 + 0.18\,\tilde{q}_{\min}}. \tag{7}$$

The minimal allowed ISCO radius is then given by

$$r_{\rm ISCO}^{\min} = 6 - 2j_{\min}\sqrt{2/3}, \tag{8}$$

and ISCO frequency corresponding to this radius is

$$\nu_{\rm ISCO}^{\min} \approx \frac{2200 M_\odot}{M}\left[1 + 0.41j_{min} + 0.49j_{min}^2\right]. \tag{9}$$

This frequency is however not the highest allowed ISCO frequency for given $\tilde{q}$. For a fixed orbital radius and q, there is $\partial\nu_K/\partial j > 0$. The maximum of $\nu_{\rm ISCO}$ for a given value of $\tilde{q}$ therefore occurs for somewhat higher value of j given by relation (6),

$$j_{\max} \approx -\frac{0.375}{0.78 - 0.23\tilde{q}_{\min}}. \tag{10}$$

The corresponding ISCO radius is now given by

$$r_{\rm ISCO}^{\max} = 6\left[1 - 0.25j_{max} + 0.38j_{max}^2\right], \tag{11}$$

and maximal ISCO frequency can be expressed as

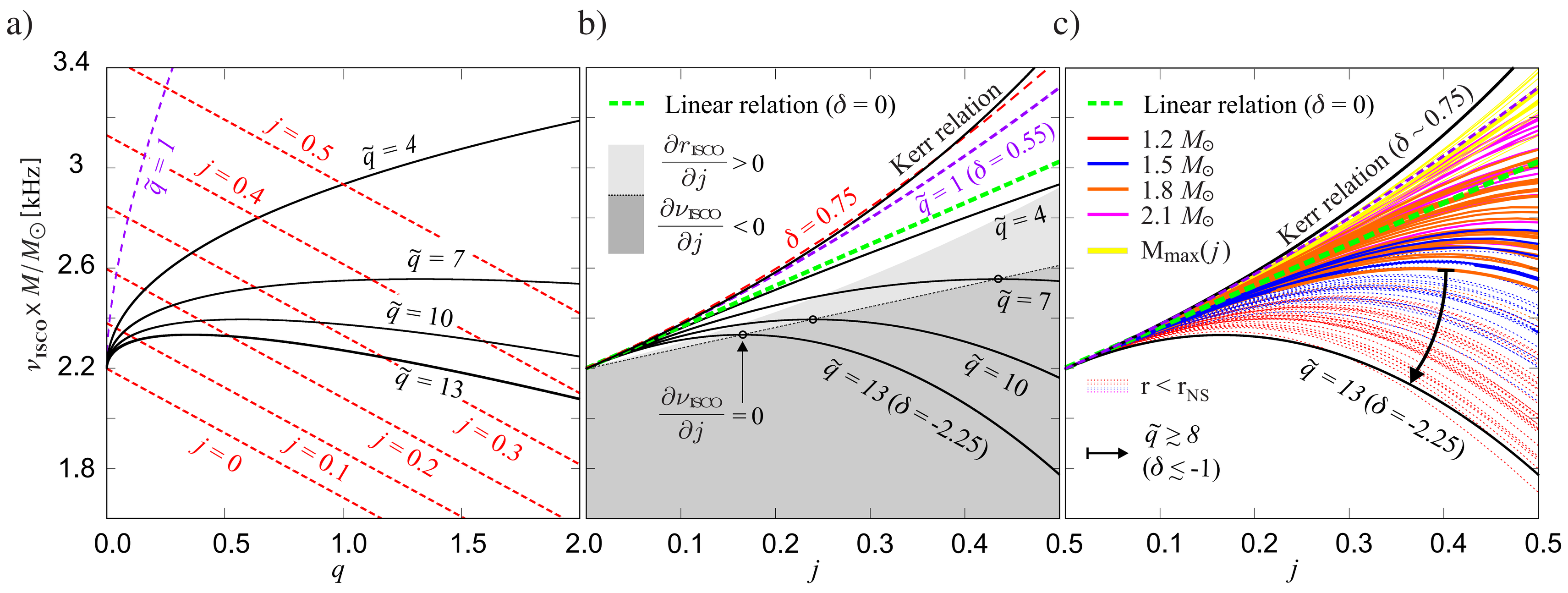
$$\nu_{\rm ISCO}^{\max} = \frac{2200 M_\odot}{M}\left[1 - \left(\frac{11\sqrt{6}}{72}\right)^2\frac{1}{\delta}\right] \approx \frac{2200 M_\odot}{M}\left[1 - \frac{0.375^2}{0.78 - 0.23\tilde{q}}\right] = \frac{2200 M_\odot}{M}\left[1 + 0.375j_{\max}\right]. \tag{12}$$

The maximal allowed ISCO frequency is therefore given by the linear function of j. We note that this limit exists only when $\tilde{q} \gtrsim 3.4$. In Figure 1b we denote the regions in the j–$\nu_{\rm ISCO}$ plane corresponding to $\partial r_{\rm ISCO}/\partial j > 0$ and $\partial\nu_{\rm ISCO}/\partial j < 0$.

References

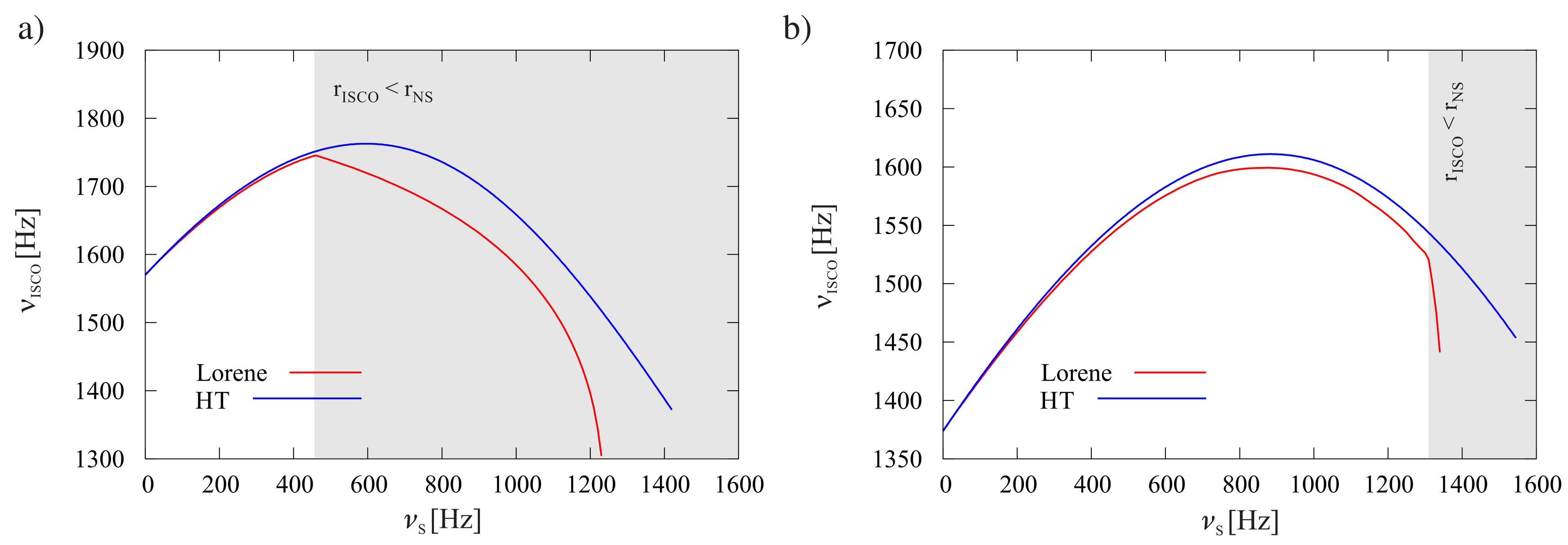
Abramowicz M. A., Almergren G. J. E., Kluzniak W., Thampan A. V., 2003, ArXiv e-prints: gr-qc/0312070
Agrawal B. K., Shlomo S., Au V. K., 2005, Phys. Rev. C, 72, 014310
Akmal A., Pandharipande V. R., Ravenhall D. G., 1998, Phys. Rev. C, 58, 1804
Arnett W. D., Bowers R. L., 1977, ApJS, 33, 415
Baldo M., Bombaci I., Burgio G. F., 1997, A&A, 328, 274
Gandolfi S., Illarionov A. Y., Fantoni S., Miller J. C., Pederiva F., Schmidt K. E., 2010, MNRAS, 404, L35
Glendenning N. K., 1985, ApJ, 293, 470
Kluźniak, W., Rosińska, D., 2014, Journal of Physics Conference Series,496, 012016
Kluzniak W., Wagoner R. V., 1985, ApJ, 297, 548
Newton W. G., Gearheart M., Li B.-A., 2013, APJS, 204,9
Rikovska Stone J., Miller J. C., Koncewicz R., Stevenson P. D., Strayer M. R., 2003, Phys. Rev. C, 68, 034324
Rikovska Stone J., Guichon P. A. M., Matevosyan H. H., Thomas A. W., 2007, Nuclear Physics A, 792, 34
Steiner A. W., Prakash M., Lattimer J. M., Ellis P. J., 2005, Physics Reports, 411, 325
Török, G., Urbanec, M., Adámek, K., Urbancová, G., A&A, 2014, 564, L5
Urbanec M., Běták E., Stuchlík Z., 2010, Acta Astron., 60, 149
Wiringa R. B., Fiks V., Fabrocini A., 1988, Phys. Rev. C, 38, 101

Figure 1



Dependence of the ISCO frequency $\nu_{\rm ISCO}$ on the NS angular momentum j and the scaled quadrupole moment $\tilde{q} = q/j^2$. a) Curves $\nu_{\rm ISCO}(q)$ drawn for constant values of j (red curves) and $\tilde{q}$ (black curves) including the Kerr limit $\tilde{q} = 1$ (purple line). b) Curves $\nu_{\rm ISCO}(j)$ drawn for constant values of $\tilde{q}$ from panel a), linear dependency (1), function $\nu_{\rm ISCO}(j)$ calculated for Kerr spacetime and relation (5) for $\delta = 0.75$. The shaded area indicates the region corresponding to the equation (6) where, in a qualitative disagreement with relativistic relation (1), the value of $\nu_{\rm ISCO}$ decreases and $r_{\rm ISCO}$ increases with increasing j. The light shadow region then indicates the region where both $r_{\rm ISCO}$ and $\nu_{\rm ISCO}$ increases with increasing j. c) Dependences $\nu_{\rm ISCO}(j)$ calculated, up to the second order in j and first order in q, for 37 EoS and four different concrete NS masses M that are colour-coded. The yellow set of curves is drawn for the case of maximal masses $M_{\max} = M_{\max}(j,\ {\rm EoS}) > 2.1M_\odot$.

Figure 2



Behaviour of $\nu_{\rm ISCO}$ (and frequency of rotation of NS surface). The exact curves (red colour) obtained using the numerical code LORENE are compared to those based on the Hartle-Thorne approximation (blue colour). For the calculation we assume the APR EoS and fixed central density ρ_c. a) Non-rotating NS mass $M_0 = 1.4M_\odot$. b) Non-rotating NS mass $M_0 = 1.6M_\odot$. The maximum of $\nu_{\rm ISCO}$ appears close to $\nu_{\rm S} = 800$Hz. However, we find that, in the case of exact curve, this maximum is not present for a fixed gravitational mass $(M = 1.6M_\odot)$. This suggest that higher-order terms in j $(j^{\rm n}, {\rm n} > 2)$ are important.

Table 1

EoS assumed in this work. The individual columns indicate the maximum mass and the corresponding radius, and the central baryon number density for each EoS along with the relevant references.

EoS	$M_{\max}^{j=0}$ $[M_\odot]$	R [km]	$n_{\rm c}$ $[fm]^{-3}$	$\mathcal{R}_{1.4}$	$M_{\max}^{\nu_{\rm S}=600}$ $[M_\odot]$	Ref.
L	2.66	13.63	0.65	5.18	2.72	Arnett & Bowers (1977)
GLENDNH3	1.96	11.38	1.05	4.90	2.00	Glendenning (1985)
SkI5	2.18	11.29	0.97	4.88	2.21	Rikovska Stone et al. (2003)
SV	2.38	11.95	0.80	4.78	2.42	Rikovska Stone et al. (2003)
N	2.63	12.77	0.72	4.77	2.68	Arnett & Bowers (1977)
SkI2	2.11	11.00	1.03	4.70	2.14	Rikovska Stone et al. (2003)
Gs	2.08	10.77	1.08	4.58	2.11	Arnett & Bowers (1977)
l	1.92	11.33	1.04	4.48	1.96	Urbanec et al.(2010)
SGI	2.22	10.93	1.01	4.46	2.25	Rikovska Stone et al. (2003)
QMC700	1.95	12.57	0.61	4.45	2.02	Rikovska Stone et al. (2007)
O	2.38	11.51	0.89	4.43	2.41	Arnett & Bowers (1977)
nocross	2.39	12.48	0.67	4.42	2.44	Urbanec et al. (2010)
J35L80	2.05	10.50	1.13	4.38	2.08	Newton et al. (2013)
UBS	2.20	12.08	0.67	4.36	2.24	Urbanec et al. (2010)
PNML80	2.02	10.41	1.16	4.35	2.04	Newton et al. (2013)
SkO	1.97	10.27	1.19	4.31	2.00	Rikovska Stone et al. (2003)
SkT5	1.82	9.95	1.31	4.23	1.84	Rikovska Stone et al. (2003)
SkO'	1.95	10.06	1.24	4.19	1.97	Rikovska Stone et al. (2003)
C	1.85	9.92	1.31	4.11	1.87	Arnett & Bowers (1977)
APR	2.21	10.16	1.12	4.10	2.23	Akmal et al. (1998)
Gandolfi	2.20	9.82	1.16	4.06	2.22	Gandolfi et al. (2010)
NRAPR	1.93	9.85	1.29	4.06	1.95	Steiner et al.(2005)
SLy4	2.04	9.95	1.21	4.03	2.06	Rikovska Stone et al. (2003)
KDE0v1	1.96	9.72	1.29	3.98	1.98	Agrawal et al. (2005)
BBB2	1.92	9.49	1.35	3.84	1.94	Baldo et al. (1997)
UU	2.19	9.81	1.16	3.84	2.21	Wiringa et al. (1988)
WS	1.84	9.52	1.38	3.77	1.86	Wiringa et al. (1988)
FPS	1.80	9.27	1.46	3.75	1.82	Lorenz et al. (1993)
AU	2.13	9.38	1.25	3.59	2.14	Wiringa et al. (1988)

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