Some Properties of Orbital Motion in The Hartle-Thorne Metric

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Abstract

The external Hartle-Thorne metric properly describes the external gravitational field of rotating compact objects in general relativity. This spacetime is characterized by gravitational mass M, angular momentum J and quadrupole moment Q. We investigate properties of orbital motion around rotating neutron stars with combinations of M, J, Q based on modeling of the rotating stars using realistic equations of state of neutron star matter. We particularly focus on impact of angular momentum and quadrupole moment of the star on epicyclic motion of matter around rotating neutron stars. This serves as a toy model for accretion disks and their oscillation frequencies. Our research is motivated by the X-ray observations of low-mass X-ray binaries where neutron star accretes matter from its binary companion and radiates in X-rays.

The Hartle-Thorne metric

The external Hartle-Thorne geometry can be expressed by using the three external parameters of the spacetime, gravitational mass M, dimensionless spin j and dimensionless quadrupole moment q in the form which can be in more detailed form found in [1]

$$
\begin{aligned}
g_{tt} &= +(1-2M/r)[1+j^2F_1(r)+qF_2(r)],\\
g_{rr} &= -(1-2M/r)^{-1}[1+j^2G_1(r)-qF_2(r)],\\
g_{\theta\theta} &= -r^2[1+j^2H_1(r)+qH_2(r)],\\
g_{\phi\phi} &= -r^2\sin^2\theta[1+j^2H_1(r)+qH_2(r)],\\
g_{t\phi} &= -2(M^2/r)j\sin^2\theta. \qquad (1)
\end{aligned}
$$

For $j = 0$ and $q = 0$ the Hartle-Thorne geometry reduces to the standard Schwarzschild geometry. The external Kerr geometry taken up to second order in angular momentum $a = Mj$ in the standard Boyer-Lindquist coordinates can be obtained from external Hartle-Thorne geometry, if we put $q = j^2$ and make the coordinate transformations

$$
r_{\mathrm{BL}} = r - \frac{a^2}{2r^3}[(r+2M)(r-M)+cos^2\theta(r-2M)(r+3M)], \qquad (2)
$$

$$
\theta_{\mathrm{BL}} = \theta - \frac{a^2}{2r^3}(r+2M)\cos\theta\sin\theta. \qquad (3)
$$

The metric coefficients of the spacetime were discussed in [2, 3, 4, 5] with attention focused on the orbital geodesic motion. For our purposes the frequencies of the near-circular epicyclic motion are relevant.

Radial profiles of the orbital frequency and epicyclic frequencies

Formulas for frequencies of circular and epicyclic motion in the Hartle-Thorne geometry has been calculated in [1] and used by many authors, see e.g. [2, 3]. Here we give the relations for the orbital (Keplerian) frequency and the radial epicyclic and vertical epicyclic frequencies that are necessary for the application of the twin HF QPO models based on the geodesic quasi-circular motion and were presented in [2].
The Keplerian frequency is given by

$$
\nu_K(r;M,j,q) = \frac{c^3}{2\pi GM}\frac{M^{1/2}}{r^{3/2}}\left[1 - j\frac{M^{3/2}}{r^{3/2}} + j^2E_1(r) + qE_2(r)\right]. \qquad (4)
$$

The radial epicyclic frequency ν_r and the vertical epicyclic frequency ν_θ are given by

$$
\nu_r^2(r;M,j,q) = \left(\frac{c^3}{2\pi GM}\right)^2\frac{(r-6M)}{r^4}[1+jF_1(r)-j^2F_2(r)-qF_3(r)], \qquad (5)
$$

$$
\nu_\theta^2(r;M,j,q) = \left(\frac{c^3}{2\pi GM}\right)^2\frac{M}{r^3}[1-jG_1(r)+j^2G_2(r)+qG_3(r)]. \qquad (6)
$$

We first demonstrate dependence of the radial profiles of the orbital and epicyclic frequencies on the dimensionless parameters j and q. For each of the frequency profile we choose the spin parameter $j = 0.1, 0.2, 0.3, 0.5$ that covers the range of the spin parameter j when one applies Hartle-Thorne approximation to astrophysically relevant models of neutron stars or strange stars [6] and stays within the limit of slow-rotation approximation that can be written as $(j/j_{\max})^2 \ << \ 1$. It has been shown that $j_{\max} \sim 0.65 - 0.7$ for neutron stars and that $j_{\max}$ for strange (quark) stars can be larger [7]. For objects, that are less compact than compact stars the value of $j_{\max}$ can be significantly larger then values for compact stars.
Values of specific quadrupole moment q are calculated assuming $q/j^2 = 1, 2, 3, 4, 5, 10$ for all considered values of j. Such a selection of parameters j and q covers astrophysically relevant situations and is obtained from modelling of the neutron stars as described in [6].

Results

The orbital frequency profiles are represented in Figure 1, the vertical epicyclic frequency profiles are represented in Figure 2, the radial epicyclic frequency profiles are represented in Figure 3. We can see that, as expected, influence of the parameters j and q/j^2 increases with decreasing radius, and at radii $r > 10M$ is relatively small for all the three frequency profiles; in all the three cases the influence of the parameter q/j^2 increases with increasing value of the dimensionless spin j. For the orbital frequency the role of the parameters j and q is smallest, and it is significantly stronger for the vertical epicyclic frequency radial profile that has similar character as the orbital frequency profiles. The strongest influence is demonstrated in the case of the radial epicyclic frequency profile – the shift of the innermost stable circular orbit corresponding to the radius where the radial epicyclic frequency vanishes is significantly shifted even for spin $j = 0.1$, and for $j = 0.3$ the radius is shifted from $r_{ms} \sim 5M$ for $q/j^2 = 1$ to $r_{ms} \sim 6M$ for $q/j^2 = 10$, and in the extreme case of $j = 0.5$, the shift is from $r_{ms} \sim 4.4M$ for $q/j^2 = 1$ to $r_{ms} \sim 6.6M$ for $q/j^2 = 10$. Clearly, the role of the quadrupole moment is most profound in the behavior of the radial epicyclic motion, as also the maximal radial epicyclic frequency strongly depends on j and q/j^2.

*

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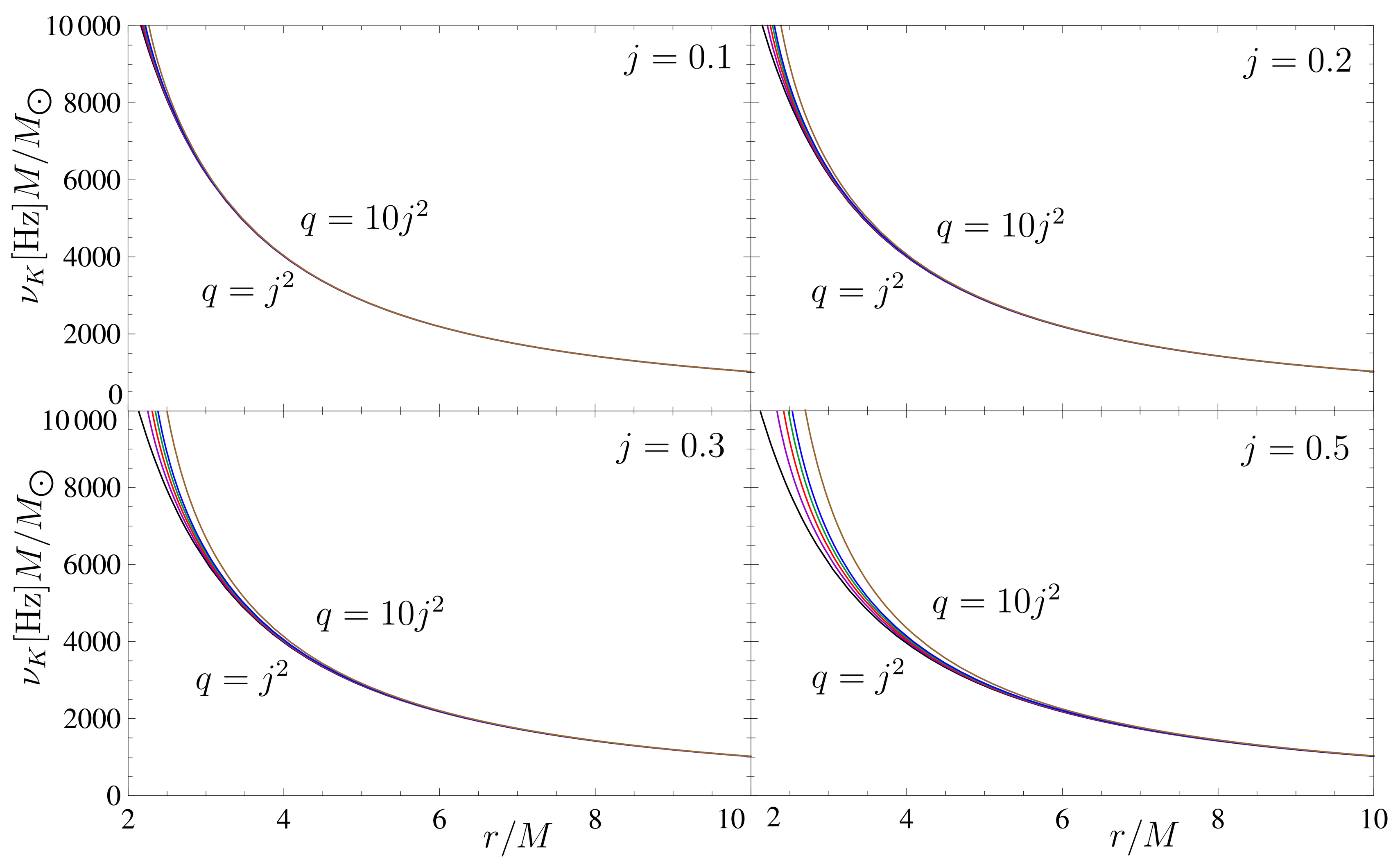
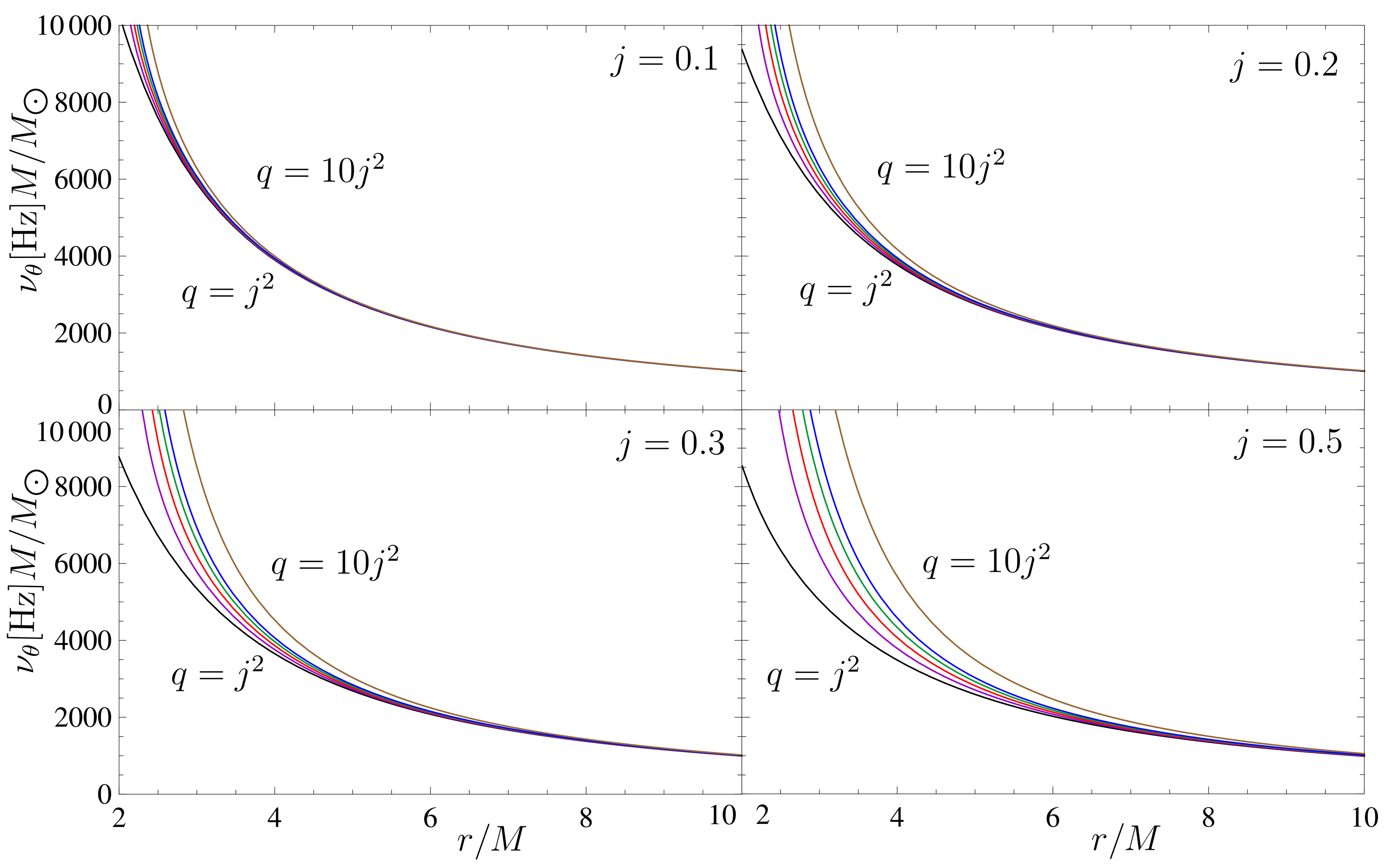
Figure 1



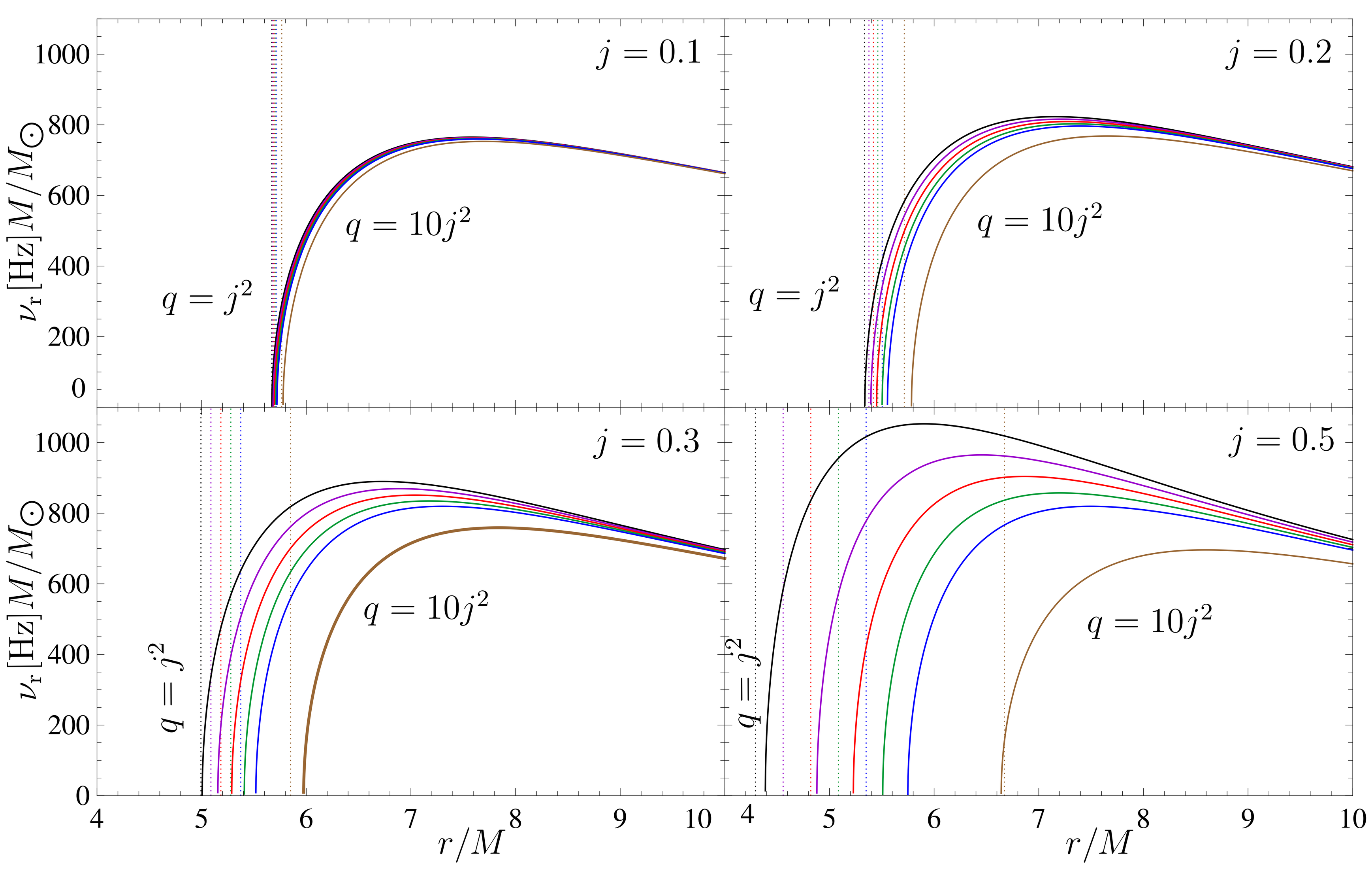
The dependence of the orbital frequency ν_K on the radial coordinate r.

Figure 2



The dependence of the vertical epicyclic frequency ν_θ on the radial coordinate r.

Figure 3



The dependence of the radial epicyclic frequency ν_r on the radial coordinate r. The vertical dotted lines are calculated values of r_{ms} for each q/j^2.

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