Simple Formula Relating Frequencies of Twin-Peak Quasiperiodic Oscillations

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Abstract

Twin-peak quasiperiodic oscillations are observed in several low-mass X-ray binary systems containing neutron stars. Timing analysis of X-ray fluxes of more than dozen of such systems reveals remarkable correlations between frequencies of two characteristic peaks present in the power density spectra. Several attempts to model these correlations with simple geodesic orbital models or phenomenological relations have failed in the past. We find and explore a surprisingly simple analytic formula that well reproduces individual correlations for a large group of sources. We also discuss a possible theoretical interpretation of this formula.

Observational Data and their Models

Twin-peak quasiperiodic oscillations (NS HF QPOs) are observed in several low-mass X-ray binary systems containing neutron stars (NSs). Timing analysis of X-ray fluxes of more than dozen of such systems reveals remarkable correlations between frequencies of two characteristic peaks present in the power density spectra. The individual correlations clearly differ, but they roughly follow a common individual pattern. This is illustrated in the left panel of Figure 1 which displays twin-peak QPOs observed in 14 different sources. A list of individual sources along with appropriate references is given in Table 1. Apart from the datapoints, we include in the left panel of Figure 1 a curve which indicates trend predicted by the relativistic precession model of HF QPOs (RP model).
Several attempts to model the individual observed correlations with simple geodesic orbital models or phenomenological relations have failed in the past. In several particular cases, fits are reliable when two free parameters specific for each source are considered, although there are still numerous clear deviations of data from the expected trend. In the right panel of Figure 1 we illustrate coefficients obtained for several three two-parametric fitting relations. These are namely the linear relation, $\nu_U = a\nu_L + b$, the quadratic relation, $\nu_U = a\nu_L^2 + b$, and the square-root relation, $\nu_U = a\sqrt{\nu_L} + b$. Inspecting this Figure one can speculate that individual frequency correlations within a large group of sources can be described by the means of a single parameter.

One-Parametric Relation

A series of works (Török et al., ApJ 2010, 2012, 2016) discusses the effective degeneracy between parameters of several orbital QPO models. Within this degeneracy, each combination of NS mass M, angular momentum j and quadrupole moment q corresponds to a certain value of a single generalized parameter $\mathcal{M}$, e.g., non-rotating NS mass. It follows that, when these parameters dominate and only non-geodesic effects that do not vary much across different systems are assumed within a given QPO model, one may expect that individual correlations can be described by a one-parametric relation,

$$\nu_{L,U} \propto (\mathrm{r},\, \mathcal{M}) \Rightarrow \nu_L = \nu_L(\nu_U,\, \mathcal{M})\,,\; \nu_U = \nu_U(\nu_L,\, \mathcal{M})\,. \tag{1}$$

This expectation is in good agreement with the possible degeneracy of two-parametric frequency relations mentioned above.

Simple Formula

In Török et al. (ApJ, 2010) we introduced a toy non-geodesic QPO model matching data of the atoll source 4U 1636-53. Motivated by results obtained therein, we attempt to describe the observed correlations by the following simple one-parametric relation:

$$\nu_L = \nu_U\left(1 - 0.8\sqrt{1 - 0.0059\,(\nu_U\mathcal{M}/\mathrm{Hz})^{2/3}}\right). \tag{2}$$

We note that this correlation scales as $\nu \propto \mathcal{M}^{-1}$ and frequency pairs $[\nu_L,\, \nu_U]$ calculated for a certain value of $\mathcal{M} = \mathcal{M}_1$ can be recalculated for another value, $\mathcal{M}_2$, using a simple multiplication

$$[\nu_L,\, \nu_U]_2 = [\nu_L,\, \nu_U]_1 \times (\mathcal{M}_1/\mathcal{M}_2). \tag{3}$$

Matching Data of Individual Sources

Assuming formula (2) we attempt to reproduce twin-peak QPO frequencies shown in the left panel of Figure 1. The results are illustrated in Figure 2. We can see that the trend observed in each of the 12 sources included within the first 8 sub-panels of Figure 2 is well matched (although there is some scatter of datapoints around the expected curves). These 12 sources span the range of $\nu_L \in (\sim 200, \sim 900)$Hz and include the atoll source 4U 1915-05 which covers a large range of frequencies, $\nu_L \in (\sim 200, \sim 800)$Hz. The two sources, GX 5-1 and GX 340+0, reveal clear deviations of data from the expected trend (see the last sub-panel of Figure 2). In Table 1 we list relevant χ^2 values along with the values of $\mathcal{M}$ parameter obtained for each of the 14 considered sources. The obtained $\mathcal{M}$ parameter values range from $\mathcal{M} = 0.7$ to $\mathcal{M} = 2.6$ while in most cases there is $\mathcal{M} \in (1.6,\, 1.9)$. These results are summarized in Table 1.

Table 1

List of sources and results of data matching. The individual columns displaying χ^2 values correspond to our simple relation (2), RP model and the cusp torus model. For these two models we assume a non-rotating neutron star mass as a free parameter (we note that the spin consideration does not improve the fits).

Source No.	Name	Type[a]	$\mathcal{M}$	χ^2	χ^2_{RP}	χ^2_{CUSP}	Number of Datapoints
(1)	4U 0614+09	A	$1.70^{\pm0.02}$	63	196	40	13
(2)	4U 1608-52	A	$1.79^{\pm0.01}$	23	92	21	12
(3)	4U 1636-53	A	$1.70^{\pm0.01}$	50	354	72	22
(4)	4U 1728-34	A	$1.56^{\pm0.01}$	49	95	39	15
(5)	4U 1735-44	A	$1.69^{\pm0.01}$	18	37	10	8
(6)	4U 1820-30	A	$1.80^{\pm0.01}$	222	590	142	23
(7)	4U 1915-05	A	$1.58^{\pm0.03}$	3	142	50	5
(8)	IGR J17191-2821	A	$1.57^{\pm0.02}$	2	3	2	4
(9)	XTE J1807.4-294	P	$2.60^{\pm0.11}$	5	9	$-^b$	7
(10)	Sco X-1	Z	$1.81^{\pm0.01}$	40	1002	90	39
(11)	Cir X-1	Z	$0.74^{\pm0.10}$	12	13	$-^b$	11
(12)	GX 17+2	Z	$1.88^{\pm0.03}$	10	51	8	10
(13)	GX 340+0	Z	$-^c$	46	20	$-^b$	12
(14)	GX 5-1	Z	$-^c$	334	61	$-^b$	21

[a] A - atoll, Z - Z, P - pulsar. [b] The observed frequencies extend below the expected range of physical applicability of the cusp torus model. [c] There is no reliable match by relation (2). The $\mathcal{M}$ parameter cannot be evaluated.

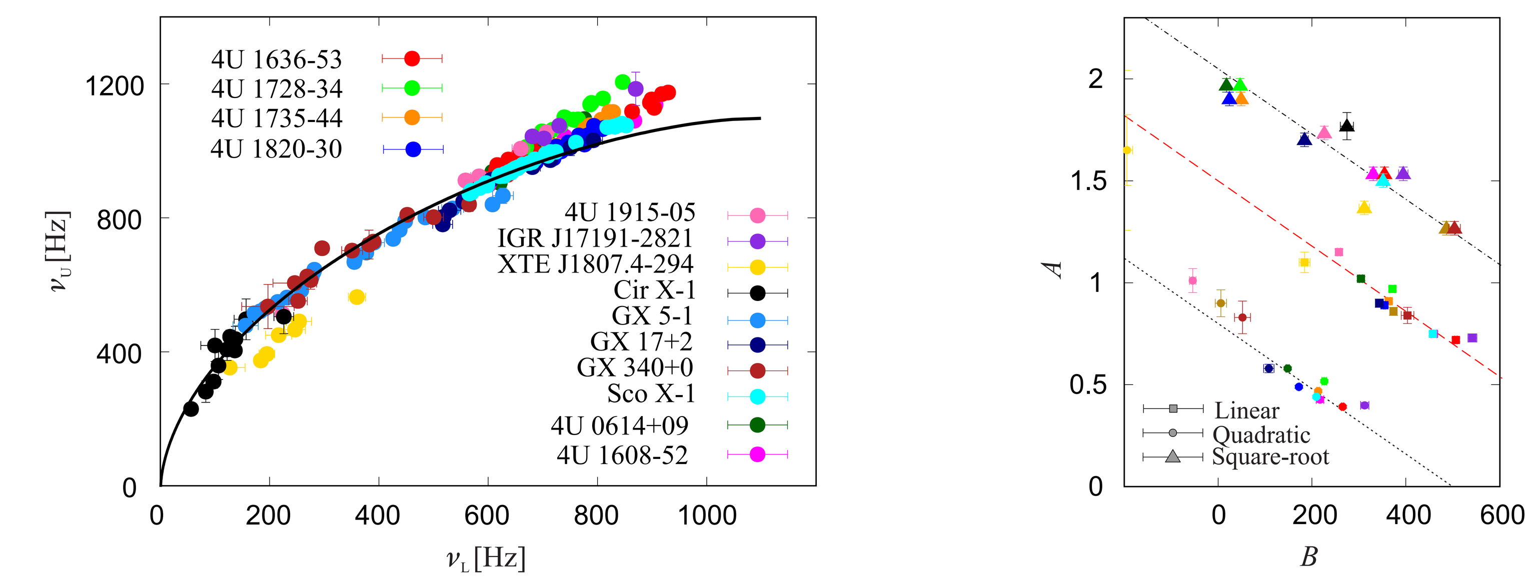
Possible Physical Interpretation

Formula (2) well fits the frequencies of twin-peak QPOs within a large group of 12 sources. This match might be of high importance for the twin-peak QPO model identification. Moreover, the frequency scaling (3), $\nu \propto \mathcal{M}^{-1}$, further supports the hypothesis of the orbital origin of NS HF QPOs since the frequencies of orbital motion scale with the NS mass M as $\nu \propto M^{-1}$. We suggest that this finding represents NS analogy of the $1/M$ scaling of the 3:2 BH HF QPO frequencies.

Oscillating Tori

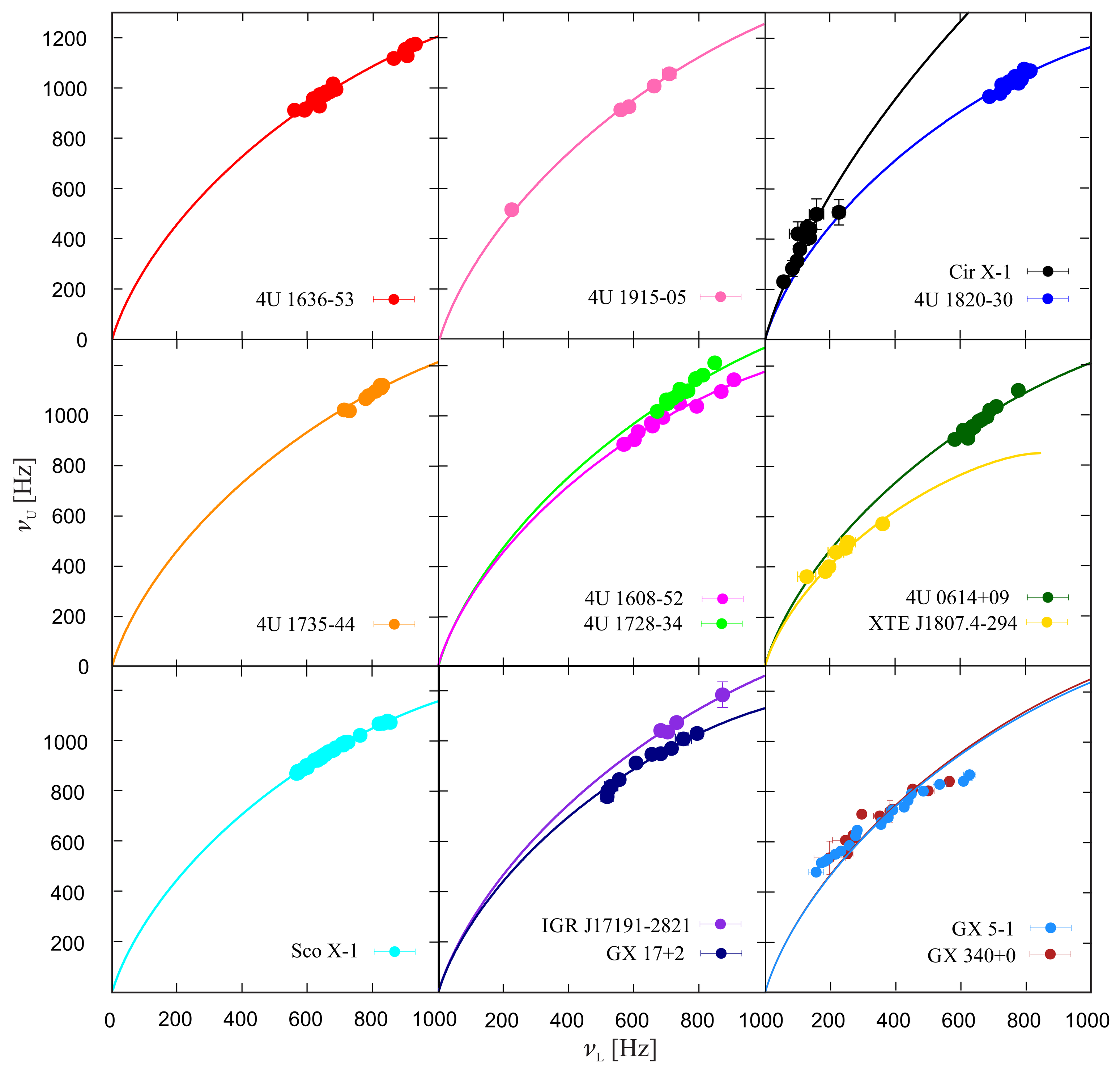
It can be shown that relation (2) well approximates the relation based on the model of an oscillating torus with cusp (see poster of Šrámková et al., HEAD 2017; Török et al., MNRAS Letters 2016). This is illustrated in the left panel of Figure 3.
We verify that for each of sources 1-12 the exact cusp torus model allows fits comparable to those obtained for relation (2). In Table 1 we include relevant illustration given by the comparison of χ^2 values of the fits by relation (2), RP model, and cusp torus model. In some cases, the interpretation of results implied by the cusp torus model is however questionable since the observed frequencies extend below the expected range of physical applicability of the model. Moreover, in the case of sources 13 and 14 (GX 5-1 and GX 340+0) we do not obtain good match for relation (2). Within the torus model framework, the data of these two sources are better described by the slender torus limit (which corresponds to reliable fits with formulae of the RP model).
Despite the above queries we conclude that the simple relation (2) not only well reproduces the data of the 12 sources, but its form may also indicate that twin-peak QPOs represent a signature of global modes of accreted fluid motion in strong gravity.

Figure 1



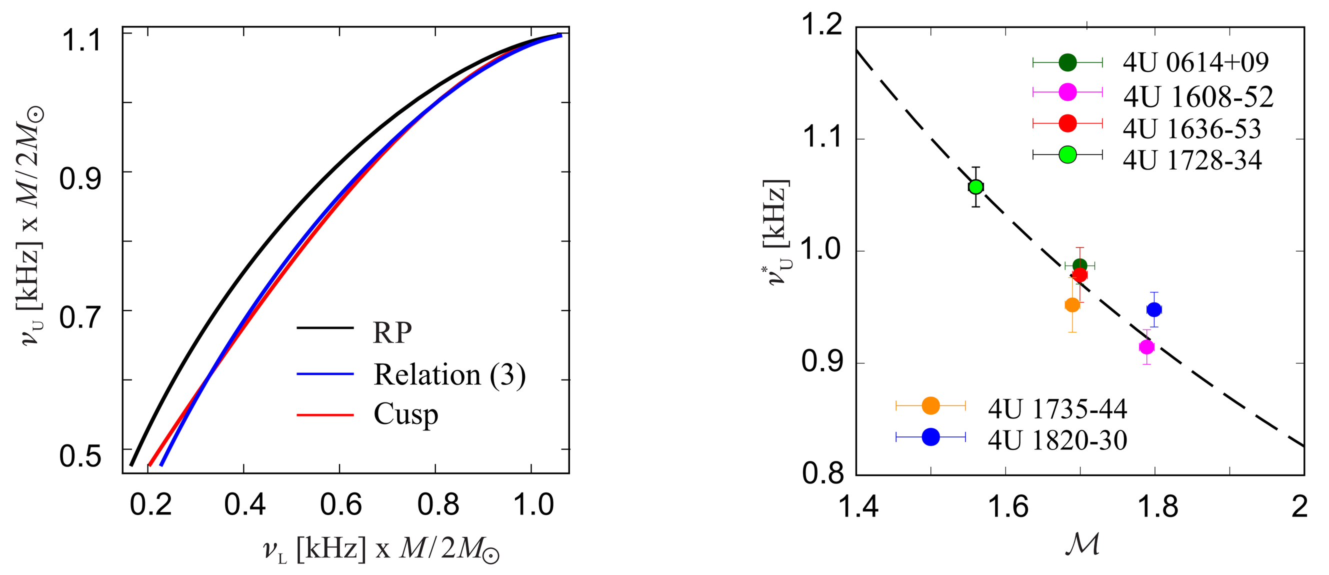
LEFT: Correlations between the frequencies of twin-peak QPOs. a) Frequencies of twin-peak QPOs in 14 sources. The black curve indicates the prediction of the RP model assuming a non-rotating NS with $M = 2M_\odot$.RIGHT: Parameters of various relations fitting the frequencies displayed in panel a) and the slope intercept anticorrelation found by Abramowicz et al. (red line). For the sake of clarity in drawing, we rescale the parameters of the individual relations as follows. Linear: $A = a,\ B = b/Hz$; quadratic: $A = (a - 500)Hz,\ B = (694\,b + 0.06)/Hz$; square-root: $A = (a + 400)/Hz^{1/2},\ B = (b/30 + 0.2)/Hz$.

Figure 2



Correlations between the twin-peak QPO frequencies in the individual sources vs. the one-parametric relation (2). Relevant values of the $\mathcal{M}$ parameter are listed in Table 1.

Figure 3



LEFT: Correlation (2) vs. correlations predicted by the standard RP model and the cusp torus model. RIGHT: Frequency values corresponding to the equality between rms amplitudes of the upper and lower QPO vs. $\mathcal{M}$ parameter in the six atoll sources. The dashed black curve corresponds to the scaling relation (4).

Other Signatures of $1/\mathcal{M}$ Scaling

It can be fruitful to further investigate possible relations between the $\mathcal{M}$ parameter and the behaviour of individual sources and their QPOs. For instance, as discussed by Török (A&A, 2009), there is a qualitative change of the relation between twin-peak QPO amplitudes (A_L and A_U) in six atoll sources, which seems to be related to the 3:2 frequency ratio. For the bottom part of the frequency correlation $\nu_U(\nu_L)$, there is $A_L < A_U$. When the QPO frequency increases from this part of the correlation towards higher frequencies, the difference between the QPO amplitudes decreases. For a certain critical frequency ν_U^* the two amplitudes are equal. In contrast to the bottom part of the correlation, within the most of its top part, there is $A_L > A_U$. We plot the critical frequency ν_U^* for each of the six sources in the right panel of Figure 3. Remarkably, this frequency scales as

$$\nu_U^* = \frac{1651}{\mathcal{M}}\,[\mathrm{Hz}]. \tag{4}$$

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